Note Title 3/29/2012

MOTION PLANNING

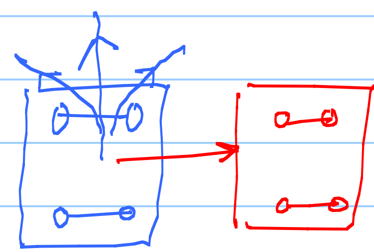
WITH KINEMATIC CONSTRAINTS

(NON-HOLONOMIC ROBOTS)

Constraints on velocities at a given q

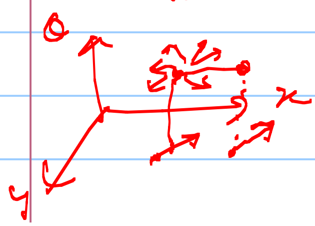
Chapter 9 Latombe

Physical example: Car like robot



① Does this constraint restrict the C-space that is reachable ??

x, y, θ



② How do we develop planners?

holonomic constraints

$q = (q_1, \dots q_m)$

Definitions :

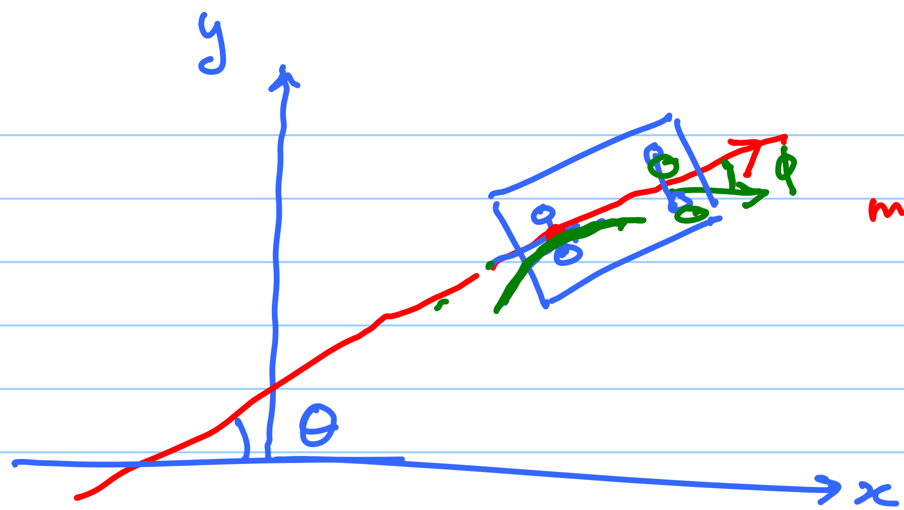
$$k \left\{ \begin{aligned} f_1(q) &= 0 \\ f_2(q) &= 0 \\ f_3(q) &= 0 \\ &\vdots \end{aligned} \right.$$

Implicit func. : $\Rightarrow$ $(m-k)$ dim space

integrable $g_1(q) = 0$

$$\left\{ f_1(q, \dot{q}) = 0 \right.$$

not integrable $\Rightarrow$ non-holonomic constraints



$$|\phi| \le \phi_{max} < \frac{\pi}{2}$$

min Curvature $= \dfrac{L}{\tan \phi_{max}}$

$\downarrow$

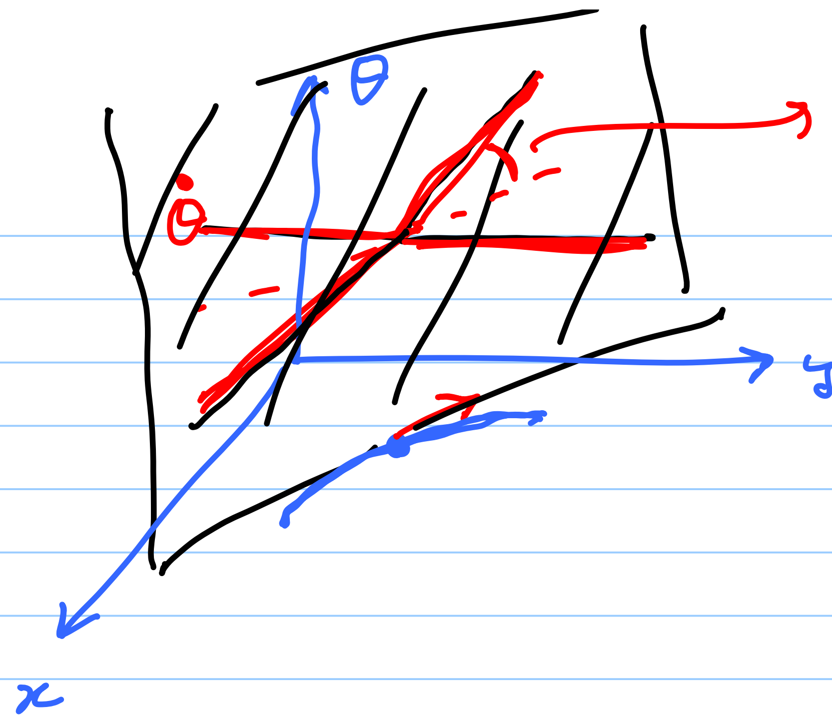
$\dfrac{1}{\rho_{min}}$

$$\frac{\dot{y}}{\dot{x}} = \tan\theta$$

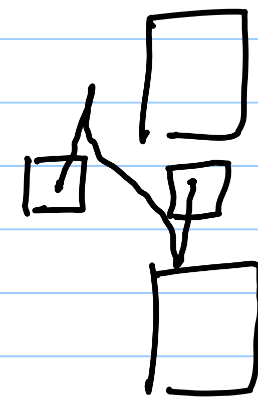
Linear in vel. variables

$$\boxed{\Leftrightarrow \; -\dot{x}\sin\theta + \dot{y}\cos\theta = 0}$$

rolling constraints $\Vert$ space of acheivable vel. at a given (x, y, θ) is a 2-dim plane.



$\tan^{-1} \frac{1}{P_{min}}$



$q = (q_1, \dots q_m)$

Char. of non-holonomic Constraints

$$\sum_{i=1}^{m} \omega_i(q)\, \dot{q} = 0$$

Frobenius integrability theorem :

for any $i, j, k \in [1, m]$

$1 \leq i < j < k \leq m$

Constraint is integrable iff

$$A_{ijk} = \omega_i \left(\frac{\partial \omega_k}{\partial q_j} - \frac{\partial \omega_j}{\partial q_k} \right) + \omega_j \left(\frac{\partial \omega_i}{\partial q_k} - \frac{\partial \omega_k}{\partial q_i} \right)$$

$$+\omega_k \left(\frac{\partial \omega_j}{\partial q_i} - \frac{\partial \omega_i}{\partial q_j} \right) = 0$$

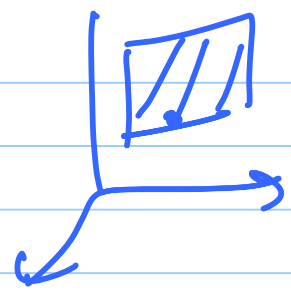
$\Longrightarrow$ If $A_{ijk} \neq 0 \Rightarrow$ Constraint is non-holonomic.

for Car like robot example: "show that the Constraint is non-holonomic"

$q = (x, y, \theta)$

$\dot{q} = (\dot{x}, \dot{y}, \dot{\theta})$

exercise



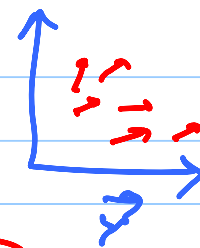
Control space: set of allowed velocities at a given q

<u>Controllability</u>: if $\exists$ a free path bet. two configs, then there also exists a free path that satisfies the constraint.

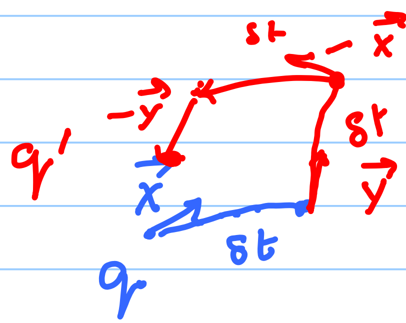
Choset's

Ex. 29

vector field:



$\alpha \vec{x} + \beta \vec{y}$



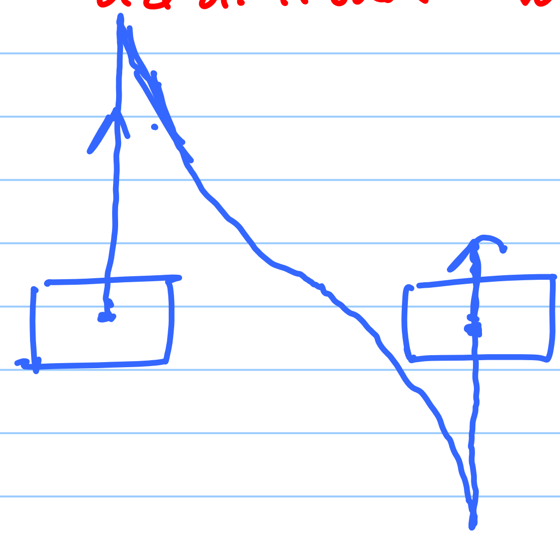
$$\lim_{\delta t \to 0} q - q' = \delta t^2 \overbrace{\left[D\vec{y} \cdot \vec{x} - D\vec{x} \cdot \vec{y} \right]}^{\text{Lie bracket}}$$

$$D\vec{y} = \begin{pmatrix} \frac{\partial y_1}{\partial q_1} & \frac{\partial y_1}{\partial q_2} & \cdots & \frac{\partial y_1}{\partial q_m} \\ \vdots & & & \\ \frac{\partial y_m}{\partial q_1} & & \cdots & \frac{\partial y_m}{\partial q_m} \end{pmatrix}$$

If Lie bracket motion is not spanned by $\vec{x} + \vec{y}$

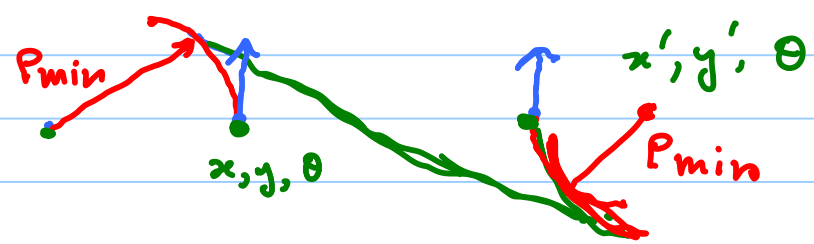
then the Lie Bracket gives you

"additional motion"

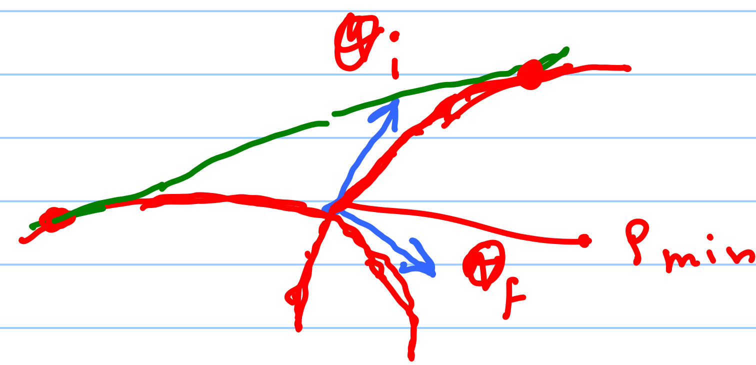


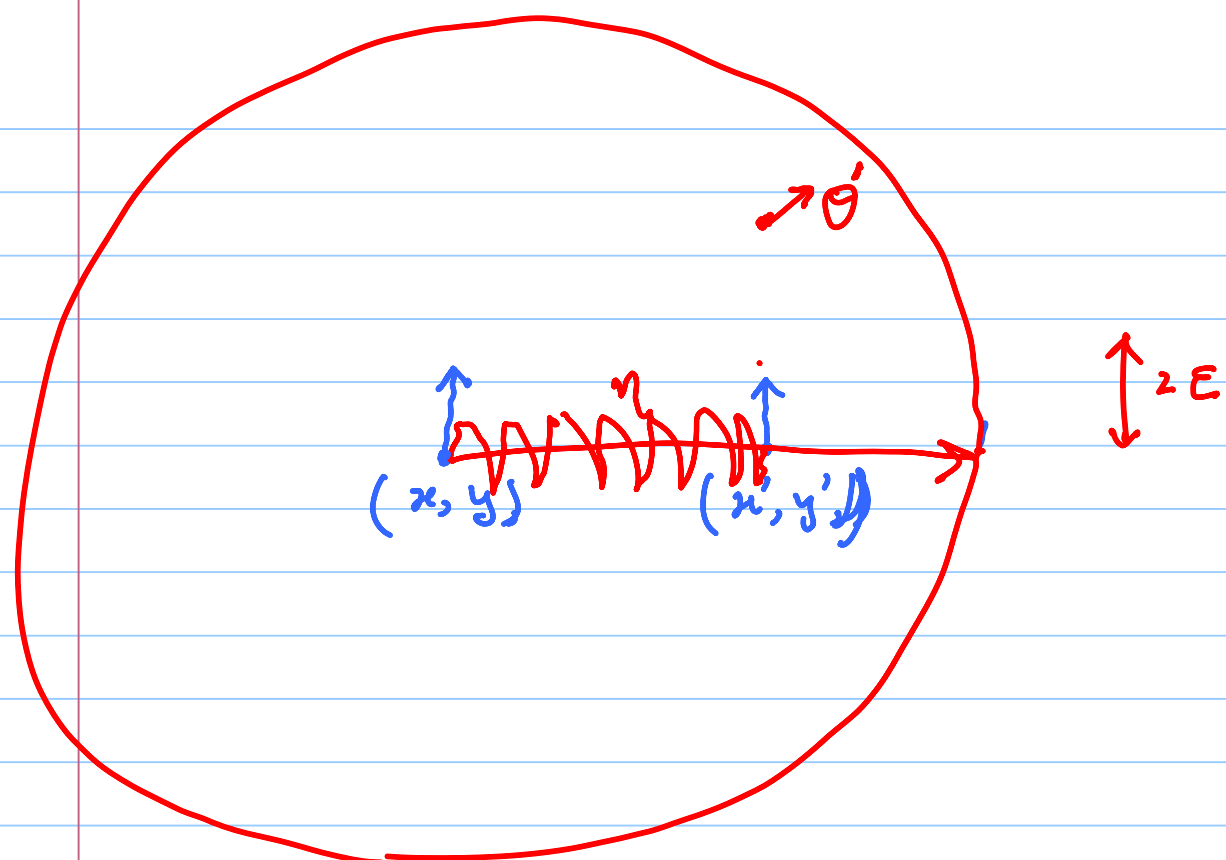
CAR-LIKE ROBOT : ρ_{min}

Man ① : side ways



Man ②



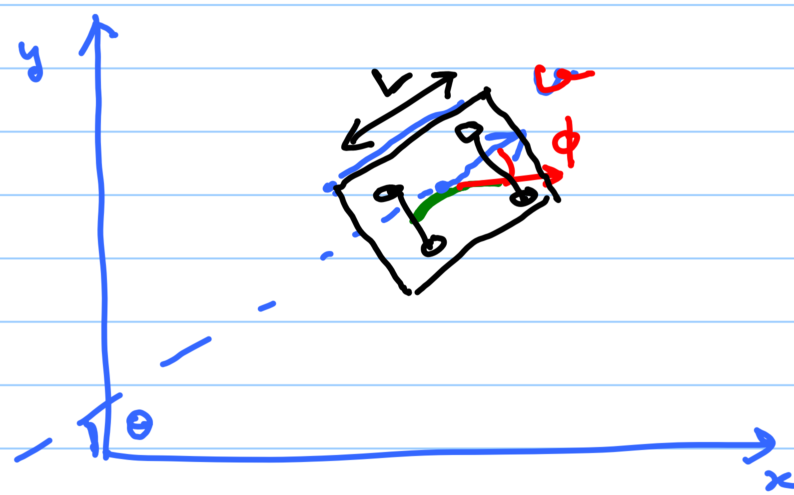


$$\vec{X}_1 = \begin{bmatrix} \cos\theta & \sin\theta & 0 \end{bmatrix}$$

$$\vec{X}_2 = \begin{bmatrix} \cos\theta & \sin\theta & \frac{1}{\rho_{min}} \end{bmatrix}$$

$$\vec{X}_3 = \begin{bmatrix} \cos\theta & -\sin\theta & -\frac{1}{\rho_{min}} \end{bmatrix}$$

Non-holonomic planner
for a car like robot



$$\dot{\theta} = \frac{v}{L} \tan\phi$$

$$\boxed{\text{Curv} = \frac{L}{\tan\phi}}$$

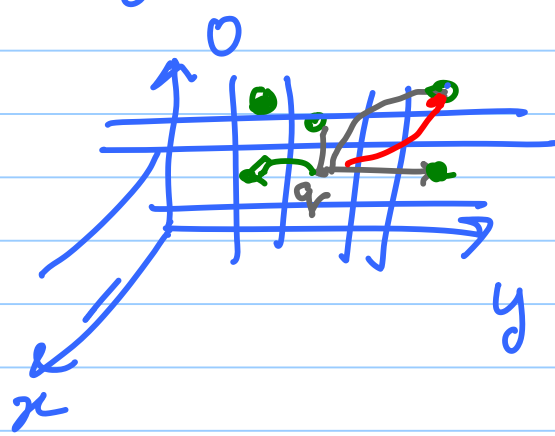
Eqns of motion:

$$\begin{cases} \dot{x} = v \cos\theta \\ \dot{y} = v \sin\theta \end{cases}$$

$$\dot{\theta} = \frac{v}{L} \tan\phi$$

numerical integration over a certain "δt"

① Discretize the c-space : x, y, θ



"cells"

② Discretize control space : v : $[v_{max}, 0, -v_{max}]$

ϕ : $[\phi_{max}, 0, -\phi_{max}]$

OPEN LIST, closed list of cells

<u>pseudo code:</u> q_{INIT}, q_{GOAL}

Put q_{init} in OPEN

DO UNTIL OPEN IS EMPTY

1. select first cell $\in$ OPEN. Call it q.
2. PUT q in closed
3. determine all successors of q and put them in SuccLIST

   apply all possible controls, use forward integration over δt

4. do until SuccLIST Is EMPTY

   pop the first element. Call it q'

determine the cell that q' belongs to

if cell $\in$ closed do nothing

else add cell to open list

if $q_{goal} \in$ cell exit.

end do

exponential in number of controls : m controls with ℓ levels each

$\ell \times \ell \times \cdots \ell$

$= \ell^m$

$$\dot{x} = v \cos\theta$$

$$\dot{y} = v \sin\theta$$

$$\dot{\theta} = \frac{v}{L} \tan\phi$$

v, ϕ

$$\theta(\cancel{t}\,\delta) = \theta(0) + \left(\frac{v}{L} \tan\phi\right) \delta t$$

$$x(\cancel{t}\,\delta) = \int v \cos\theta \, dt$$

$$= v \int \cos\theta \, d\theta \cdot \frac{dt}{d\theta}$$

$$= v \int \cos\theta \, d\theta \; \frac{L}{v \tan\phi}$$

$$= \frac{L}{\tan\phi} \int \cos\theta \, d\theta$$

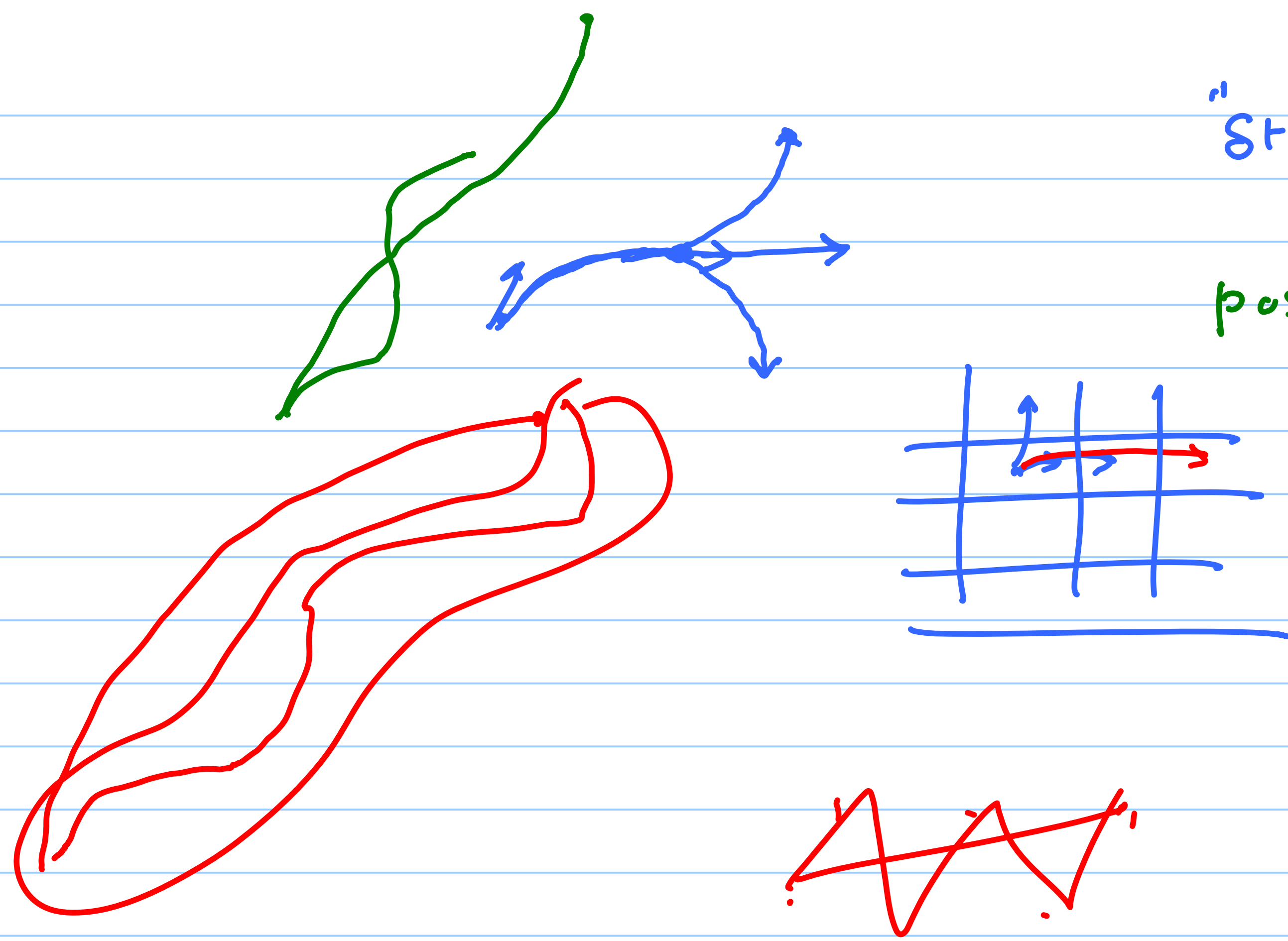
$$= \frac{L}{\tan\phi}\left[\sin\theta_{\delta} - \sin\theta_0\right]$$

missing in text

$$x(\delta t) = x(0) + \frac{L}{\tan\phi}\left[\sin\left[\theta(0) + \left(\frac{u}{L}\tan\phi\right)\delta t\right] - \sin\theta(0)\right]$$

$$y(\delta t) = y(0) - \frac{L}{\tan\phi}\left[\cos\left(\theta(0) + \frac{u}{L}\tan\phi\,.\,\delta t\right) - \cos\theta(0)\right]$$

Look at fig. 9, 10, 11, 12



"δt" critical choice

post processing needed to make paths "shorter"

Dates :

1) No Lecture on Tuesday April 10^{th}

2) Take home final handed
due April 16^{th} (hard copy)

3) project demo 23^{rd}
&
report (5 pages or so) $\rightarrow$

4) April 27^{th}

Report → 5-6 pages

1. project goal / objective
2. outline of your overall approach. description of main algorithms
3. pseudo code of main and various subalgorithms
4. samples / run times → experiments
5. lessons learned / pros + cons. of your planner